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An oligopoly-fringe non-renewable resource game in the presence of a renewable substitute☆

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ABSTRACT

In accordance with recent empirical evidence, we model the oil market as an oligopoly facing a fringe as well as competition from renewable resources. Within this framework we fully characterize, i.e., for all vectors of initial resource stocks, the equilibrium extraction paths of the fringe and the oligopolists. We show that (i) the sequence of extraction in equilibrium crucially depends on the oligopolists' market power, (ii) there always exists a phase of simultaneous supply of the oligopolists and the fringe, (iii) the oligopolists pursue a limit-pricing strategy near the end of the extraction horizon, and (iv) an increase in the reserves of the fringe may lead to a *decrease* in their initial supply.

1. Introduction

In this paper we build a model that incorporates three important features of the global oil market: the existence of a number of big suppliers with market power, a competitive fringe and the presence of a renewable substitute. We allow the degree of imperfect competition to vary by considering an arbitrary number of suppliers with market power. We fully characterize the equilibrium in such a market.

Our paper is related to the literature on non-renewable resource extraction under imperfect competition. Seminal contributions to this research area were made by Stiglitz (1976) and Stiglitz and Dasgupta (1982) on monopoly, Lewis and Schmalensee (1980) and Salo and Tahvonen (2001) on oligopoly, and Gilbert (1978) and Newbery (1981) on dominant firms. This literature has been extended to a cartel-fringe setting by Salant (1976), Groot et al. (2003), Benchekroun et al.

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(2010) and Benchekroun and Withagen (2012), and to an asymmetric oligopoly by Benchekroun et al. (2009). In a closely related paper, we have characterized a cartel-fringe equilibrium in the presence of renewables, taking into account damages from climate change (Benchekroun et al., 2017). The current article is complementary to our earlier work, as we now focus on extending the cartel-fringe equilibrium to an oligopoly-fringe setting, but leave out climate damages and do not perform a welfare analysis.

In this paper we try to replicate the market structure of the global oil market in which OPEC countries own more than 80% of the proven oil reserves and have a market share of about 40% of global oil production (cf. EIA, 2017). To do this, we follow Almoguera et al. (2011, p.144) who conclude that "OPEC's behavior is best described as Cournot competition in the face of a competitive fringe constituted by non-OPEC producers." Accordingly, our model's market structure consists of a large number of price-taking mining firms, the fringe, and a finite number of firms with market power, the oligopolists.[1]

Another important feature of our framework is the existence of renewables that supply a perfect substitute for oil at a constant marginal cost.[2] This puts an upper bound on the price of oil and raises the possibility that the equilibrium features limit-pricing by the oligopolists (see, e.g., Wang and Zhao (2013), Andrade de Sá and Daubanes (2016) and Van der Meijden et al. (2018) for recent and Hoel (1978b), Hoel (1978a), Gilbert and Goldman (1978), Salant (1979), and Stiglitz and Dasgupta (1982) for earlier work). A strategy of limit pricing entails that the oligopolists supply just enough to drive the oil price down to such an extent that renewables producers cannot profitably enter the market, i.e., they effectively set the price of oil marginally below the unit production costs of renewables.

We show, by construction, the existence of an open-loop Nash-Cournot equilibrium on the oil market. Our main findings are as follows. First, although we assume that the oligopolists have lower marginal extraction costs than the fringe, there will always be a phase of simultaneous supply by both types of producers. The reason is that oligopolists use their market power to mark up the price. This enables the fringe to start supplying before the stocks of the oligopolists are depleted.

Second, we show that the sequence of extraction phases in equilibrium is dependent on the oligopolists' market power, as measured by the number of oligopolists (where a high number of oligopolists corresponds to a low level of market power). In case of strong or intermediate market power, the equilibrium starts with a phase during which the oligopolists and the fringe simultaneously supply oil to the market. In case of weak market power, however, there may be an initial phase during which the oligopolists are the sole suppliers.

Third, we determine the conditions under which oligopolists will perform a limit-pricing strategy. In case of strong market power, limit pricing will commence as soon as the fringe depletes its stock. In case of intermediate or weak market power, however, oligopolists will only choose for limit pricing after the fringe's stock is depleted and their own remaining stocks have been driven down below a critical threshold.

Fourth, we show that—perhaps counterintuitively—an increase in the initial resource stock of the fringe may lead to a *decrease* in its initial extraction. The reason is that the increase in the fringe's initial resource stock provokes front-loading of extraction by the oligopolists, which crowds out initial extraction of the fringe.

In our baseline model, we impose constant marginal extraction costs. Moreover, throughout the paper we use an open-loop equilibrium concept. In Section 4, we will numerically investigate the effects of stock-dependent extraction costs of the fringe and discuss the merits and shortcomings of our open-loop compared to a closed-loop equilibrium approach.

The remainder of the paper is structured as follows. Section 2 outlines the model and characterizes the open-loop Nash-Cournot equilibrium. Section 3 discusses the effect of the oligopolists' market power on the sequence of resource extraction phases. Section 4 discusses stock-dependent marginal extraction costs and the closed-loop as an alternative equilibrium concept. Finally, Section 5 offers concluding remarks.

2. The model

We consider the market for perfectly substitutable renewable and non-renewable resources. The non-renewable resource is supplied by a group of $n < \infty$ suppliers with market power, referred to as oligopolists, and by a price-taking fringe. The fringe owns an aggregate initial stock S_0^f and extracts at constant per unit extraction cost k^f. All oligopolists have identical initial stocks $S_{0i}^o = S_0^o/n$, and identical unit extraction costs $k^o < k^f$. Extraction rates at time $t \geq 0$ by the fringe and oligopolist i are, respectively, denoted by $q^f(t)$ and $q_i^o(t)$. We write aggregate supply by the oligopolists as $q^o(t) \equiv \sum_i q_i^o(t)$. The inverse demand for the non-renewable resource at time t is linear and given by $p(t) = \alpha - \beta(q^f(t) + q^o(t))$, with $\alpha > k^f$ and $\beta > 0$. The perfect renewable substitute for the non-renewable resource can be produced at constant marginal cost $b > 0$ with $k^f < b < \alpha$. Hence, demand for the non-renewable resource vanishes for $p > b$. The prevailing interest rate r is constant.

[1] The cartel-fringe model is obtained as a special case of our framework when the number of firms with market power is set to one. The polar cases of perfect competition and monopoly, facing competition of a renewable substitute, can also be generated as special cases within our framework by respectively setting the resource stock of oligopolists or the resource stock of the fringe equal to zero and the number of oligopolists to one.

[2] We abstract from capacity build-up in the renewables sector (cf. Fischer et al., 2004; Powell and S. Oren, 1989; Tsur and Zemel, 2011; Wirl and Withagen, 2000).

2.1. Behaviour of the fringe and the oligopolists

The objective of the fringe is to maximize its discounted profits,

$$\int_0^\infty e^{-rt}(p(t)-k^f)q^f(t)dt,$$

where the price *path* is taken as given, and the extraction path satisfies its resource constraint

$$\dot{S}^f(t) = -q^f(t),\ q^f(t) \geq 0,\ S^f(t) \geq 0 \text{ for all } t \geq 0, \text{ and } S^f(0) = S_0^f.$$

Oligopolist i takes the time *paths* of q^f and q_j^o $(j \neq i)$ as given and maximizes

$$\int_0^\infty e^{-rt}\left[\alpha - \beta\left(q^f(t) + \sum_{j=1}^n q_j^o(t)\right) - k^o\right]q_i^o(t)dt,$$

subject to its resource constraint

$$\dot{S}_i^o(t) = -q_i^o(t),\ q_i^o(t) \geq 0,\ S_i^o(t) \geq 0 \text{ for all } t \geq 0, \text{ and } S_i^o(0) = S_{0i}^o.$$

The existence of perfectly substitutable renewables results in the following additional constraint

$$\alpha - \beta(q^f(t) + q^o(t)) \leq b, \tag{1}$$

since the price oligopolists can ask has b as an upper bound.

2.2. Equilibrium

An equilibrium is defined as follows.

Definition 1. A vector of functions $(p, q) \equiv (p, q_1^o, \dots, q_n^o, q^f)$, with $(p(t), q(t)) \geq 0$ for all $t \geq 0$ and $p(t) = \alpha - \beta\left(q^o(t) + q^f(t)\right)$, is an open-loop Oligopoly-Fringe Equilibrium (OL-OFE) if

(i) each extraction path of the vector q satisfies the corresponding resource constraint,
(ii) for all $i = 1, 2, \dots, n$

$$\begin{aligned}&\int_0^\infty e^{-rs}\left[\alpha - \beta\left(q^o(s) + q^f(s)\right) - k^o\right]q_i^o(s)ds \\ &\geq \int_0^\infty e^{-rs}\left[\alpha - \beta\left(\sum_{j\neq i} q_j^o(s) + \hat{q}_i^o(s) + q^f(s)\right) - k^o\right]\hat{q}_i^o(s)ds,\end{aligned}$$

for all $\hat{q}_i^o$ satisfying the resource constraint, and
(iii)

$$\int_0^\infty e^{-rs}[p(s) - k^o]q^f(s)ds \geq \int_0^\infty e^{-rs}[p(s) - k^o]\hat{q}^f(s)ds,$$

for all $\hat{q}^f$ satisfying the resource constraint.

We characterize an OL-OFE by using optimal control theory. The Hamiltonian associated with the fringe's problem reads

$$\mathcal{H}^f(t, q^f, \lambda^f) = e^{-rt}(p(t) - k^f)q^f - \lambda^f q^f.$$

The necessary conditions include

$$p(t) - k^f = \alpha - \beta(q^f(t) + q^o(t)) - k^f \leq \lambda^f(t)e^{rt}, \tag{2a}$$

$$\left(p(t) - k^f - \lambda^f(t)e^{rt}\right)q^f(t) = 0, \tag{2b}$$

$$\dot{\lambda}^f(t) = 0, \tag{2c}$$

where λ^f denotes the shadow price of the resource stock of the fringe. Hence, the conditions say that in an equilibrium with positive supply of the fringe, the net price, $p - k^f$, grows over time at a rate equal to the rate of interest. The Hamiltonian and the Lagrangian associated with oligopolist i's problem read, respectively,

$$\mathcal{H}_i^o(t, q_i^o, \lambda_i^o) = e^{-rt}\left[\alpha - \beta\left(q^f(t) + \sum_{j\neq i} q_j^o(t) + q_i^o\right) - k^o\right]q_i^o - \lambda_i^o q_i^o,$$

$$\mathcal{L}_i^o(t, q_i^o, \lambda_i^o, \mu_i^o) = \mathcal{H}_i^o + \mu_i^o\left[b - \alpha + \beta\left(q^f(t) + \sum_{j\neq i} q_j^o(t) + q_i^o\right)\right].$$

Since the oligopolists are identical, we focus on the conditions that characterize a symmetric equilibrium, namely where for all i we have $q_i^o = q^o/n$, and $\lambda_i^o = \lambda^o$, $\mu_i^o = \mu^o$. The necessary conditions include

$$\alpha - \beta\left[q^f(t) + \left(1 + \frac{1}{n}\right)q^o(t)\right] - k^o \leq \lambda^o(t)e^{rt} - \mu^o(t)\beta e^{rt}, \tag{3a}$$

$$\left(\alpha - \beta\left[q^f(t) + \left(1 + \frac{1}{n}\right)q^o(t)\right] - k^o - \lambda^o(t)e^{rt} + \mu^o(t)\beta e^{rt}\right)q^o(t) = 0, \tag{3b}$$

$$\alpha - \beta\left(q^f(t) + q^o(t)\right) - b \leq 0, \tag{3c}$$

$$\left(\alpha - \beta[q^f(t) + q^o(t)] - b\right)\mu^o(t) = 0, \tag{3d}$$

$$\dot{\lambda}^o(t) = 0, \tag{3e}$$

where λ^o denotes the shadow price of the resource stock of the oligopolists and μ^o is the Lagrange multiplier associated with restriction (1). Hence, the conditions imply that as long as $p < b$ (i.e., as long as restriction (1) is non-binding) and $q_i^o > 0$, marginal profits of the oligopolists increase over time at the rate of interest. Finally, the optimal depletion time T of the oligopolists' resource stocks follows from

$$\left(p(T) - k^o - \lambda^o e^{rT}\right)\frac{q^o(T)}{n} = 0. \tag{4}$$

2.3. Extraction phases and sequences

Four different phases of resource extraction can occur in an OL-OFE. We denote by F, O, S and L the phases during which only the fringe is supplying, only the oligopolists are supplying at a price strictly below b, the fringe and the oligopolists are supplying simultaneously, and only the oligopolists are supplying at a price b (i.e., limit pricing: the oligopolists marginally undercut the renewables producers' production costs to keep them at bay), respectively. In the sequel T^F, T^O, T^S and T^L will indicate the moments at which each of these phases comes to an end. We will show that phases do not repeat themselves, so that the meaning of these moments is unambiguous. We will use the symbol '$\rightarrow$' to indicate the ordering of phases, e.g., $S \rightarrow L$ means that a phase with simultaneous use is preceding a phase with limit pricing. We use the notation $t \in \Phi$ to refer to a moment where the phase of resource extraction is Φ with $\Phi \in \{F, O, S, L\}$.

We first characterize the price and extraction paths during each phase (Lemma 1) and then derive conditions under which specific sequences of phases can or cannot occur in equilibrium (Lemma 2–4).

Lemma 1. *For $t \in F$ we have*

$$p(t) = \alpha - \beta q^f(t), \tag{5a}$$

with

$$p(t) = k^f + \lambda^f e^{rt}, \tag{5b}$$

$$p(t) \leq k^o + \lambda^o e^{rt}, \tag{5c}$$

implying

$$q^f(t) = \frac{1}{\beta}(\alpha - k^f - \lambda^f e^{rt}). \tag{5d}$$

For $t \in S$ we have

$$p(t) = \alpha - \beta(q^f(t) + q^o(t)), \tag{6a}$$

with

$$p(t) = k^f + \lambda^f e^{rt}, \tag{6b}$$

$$p(t) = k^o + \lambda^o e^{rt} + \beta \frac{q^o(t)}{n}, \tag{6c}$$

implying

$$q^f(t) = \frac{1}{\beta}\Big(\alpha - (n+1)(k^f + \lambda^f e^{rt}) + n(k^o + \lambda^o e^{rt})\Big), \tag{6d}$$

$$\frac{q^o(t)}{n} = \frac{1}{\beta}\Big(k^f + \lambda^f e^{rt} - k^o - \lambda^o e^{rt}\Big). \tag{6e}$$

For $t \in O$ *we have*

$$p(t) = \alpha - \beta q^o(t), \tag{7a}$$

with

$$p(t) \le k^f + \lambda^f e^{rt}, \tag{7b}$$

$$p(t) = k^o + \lambda^o e^{rt} + \beta \frac{q^o(t)}{n}, \tag{7c}$$

implying

$$\frac{q^o(t)}{n} = \frac{1}{\beta}\frac{1}{n+1}(\alpha - k^o - \lambda^o e^{rt}), \tag{7d}$$

$$p(t) = \frac{1}{n+1}\Big(\alpha + n(k^o + \lambda^o e^{rt})\Big), \tag{7e}$$

For $t \in L$ *we have*

$$p(t) = \alpha - \beta q^o(t), \tag{8a}$$

with

$$p(t) = b, \tag{8b}$$

$$b - \frac{\alpha - b}{n} \le k^o + \lambda^o e^{rt}, \tag{8c}$$

implying

$$q^o(t) = \frac{\alpha - b}{\beta}. \tag{8d}$$

Proof. These conditions are obtained by applying the Maximum Principle to the problem of each oligopolist and the fringe and by exploiting symmetry. □

Although it occurs in a knife-edge case, an equilibrium may consist of simultaneous use throughout. Lemma 2 specifies the required initial stocks and duration of such an equilibrium.

Lemma 2. *Given* S_0^f, *if the equilibrium reads S throughout, then there exist* T^S *and* S_0^o *such that*

$$r\beta S_0^f = \Big(\alpha + nk^o - (1+n)k^f\Big)(rT^S - 1 + e^{-rT^S}) + [\alpha - b](1 - e^{-rT^S}), \tag{9a}$$

$$r\beta S_0^o = n(k^f - k^o)(rT^S - 1 + e^{-rT^S}). \tag{9b}$$

Proof. For $t \in S$ we have (6d) and (6e). Moreover, price continuity at the final time T^S together with (4) and (6b) implies $b = k^o + \lambda^o e^{rT^S}$ and $b = k^f + \lambda^f e^{rT^S}$, respectively. Solving the integrals $\int_0^{T^S} q^f dt = S_0^f$ and $\int_0^{T^S} q^o dt = S_0^o$ yields the result. □

Other equilibria, with different subsequent phases of extraction, are possible as well. In the same vein as done in Lemma 2, we can calculate the initial stocks and final times of these equilibria. In case of multiple subsequent phases, we also have to determine the phase switching times. Before we discuss how the possible sequences of extraction phases in equilibrium depend on initial resource stocks and on other parameter values, we can upfront exclude several possibilities, as shown in the next lemma.

Lemma 3. *In an OL-OFE the following holds:*

(i) Suppose $t_1 \in L$. *Then* $t \in L$ *for all* $t \ge t_1$.
(ii) Suppose $t_1 \in F$. *Then* $t \in F$ *for all* $t \ge t_1$.
(iii) The final regime is not O.
(iv) A direct transition from O to F is excluded.

Proof. See Appendix A.2. □

Part (i) and (ii) imply that L and F can only occur as final phases. Intuitively, during all phases but the limit-pricing phase, the resource price is increasing and lower than the price of renewables. Together with price continuity this ensures that there can be no other extraction phase after the limit-pricing phase. A phase F can only occur once the stocks of the oligopolists are depleted, because the marginal profits of the oligopolists are growing at a rate lower than the rate of interest during F. Given that the fringe will never choose to adopt limit pricing, this implies that F can only occur as a final extraction phase. Part (iii) is intuitive as well: if the final regime would be O, marginal profits of the oligopolists would jump upward at the time of depletion. Finally, a direct transition from O to F, as excluded in part (iv), would imply a downward jump in the oligopolists' extraction levels, and therefore an upward jump in their marginal profits.

The possible occurrence of phase O, during which oligopolists are the sole suppliers of the non-renewable resource at a price strictly below the renewables price, depends crucially on the marginal profit of an oligopolist, which—for each individual oligopolist—is given by

$$\Pi(q^f, q^o) \equiv \alpha - \beta\left(q^f + \frac{1+n}{n}q^o\right) - k^o. \tag{10}$$

Define $\tilde{q} \equiv (\alpha - k^f)/\beta$ and $\hat{q} \equiv (\alpha - b)/\beta$ as demand at prices k^f and b, respectively. Imposing $q^f = 0$ in (10), we can write marginal profits when the oligopolists are the sole suppliers at these prices as

$$\tilde{\Pi} \equiv \Pi(0, \tilde{q}) = -\frac{1}{n}\left[\alpha + nk^o - (1+n)k^f\right], \tag{11a}$$

$$\hat{\Pi} \equiv \Pi(0, \hat{q}) = -\frac{1}{n}[\alpha + nk^o - (1+n)b]. \tag{11b}$$

The next lemma specifies how the occurrence and timing of a phase O depend on the signs of $\tilde{\Pi}$ and $\hat{\Pi}$.

Lemma 4.

(i) *Suppose* $\tilde{\Pi} \leq 0$. *Then*
  (a) *The initial phase is not O.*
  (b) *A direct transition from O to S is excluded.*
(ii) *Suppose* $\tilde{\Pi} > 0$. *Then a transition from S to O is excluded.*
(iii) *Suppose* $\hat{\Pi} \leq 0$. *Then there is no phase O in equilibrium.*

Proof. See Appendix A.2. □

Intuitively, if $\tilde{\Pi} \leq 0$ the oligopolists will not choose to supply at a price strictly below k^f during an O phase, because then they would be able to increase profits by lowering extraction. Furthermore, it follows from (7c) that in a phase O the net price $p(t) - k^f$ would grow at a rate lower than the interest rate. As a result, the fringe would prefer to start extracting immediately. Hence, $\tilde{\Pi} \leq 0$ implies that phase O cannot occur as long as the fringe's stock is not depleted, which also ensures that a transition from O to S is impossible.

The intuition for part (ii) is that the extraction path of the oligopolists must be continuous to prevent a jump in marginal profits, as long as the resource price is strictly below the price of renewable energy. Price continuity then requires that the fringe's extraction rate tends to zero before a transition from S to O. However, if $\tilde{\Pi} > 0$, it follows from (6a) and (6e) that the fringe's net price, $p(t) - k^f$, grows at a rate larger than the rate of interest near this transition. Hence, the fringe prefers to conserve instead of extract, which implies that a transition from S to O cannot occur.

Regarding part (iii), if $\hat{\Pi} \leq 0$ the oligopolists will not choose to supply at a price strictly below b if they would be the sole suppliers, implying that the oligopolists will adopt limit-pricing as soon as the fringe's stock is exhausted.

3. Market power and extraction sequence

We are now ready to characterize the equilibrium for all vectors of stocks and all admitted parameter values. From the above analysis, the signs of the marginal profits $\tilde{\Pi}$ and $\hat{\Pi}$ will play a key role in the equilibrium outcome. These signs depend on all market and cost parameters. Here we take the number of oligopolists as a pivotal parameter, while keeping their aggregate stock S_0^o fixed. Intuitively, a larger number of oligopolists implies that each of them has less market power. As a result, for a given level of aggregate supply by the oligopolists, individual oligopolist's marginal profits increase when the number of oligopolists increases. We define the threshold levels $n_{\tilde{\Pi}} \equiv \frac{\alpha - k^f}{k^f - k^o}$ and $n_{\hat{\Pi}} \equiv \frac{\alpha - b}{b}$ as follows:

$$\tilde{\Pi} > 0 \Leftrightarrow n > n_{\tilde{\Pi}}, \tag{12a}$$

$$\hat{\Pi} > 0 \Leftrightarrow n > n_{\hat{\Pi}}. \tag{12b}$$

Since $k^f < b < \alpha$, we have $n_{\tilde{\Pi}} > n_{\hat{\Pi}}$. We will separately consider three cases:

(i) *Strong market power*: $n \leq n_{\hat{\Pi}}$ implying $\tilde{\Pi} < 0$ and $\hat{\Pi} \leq 0$.

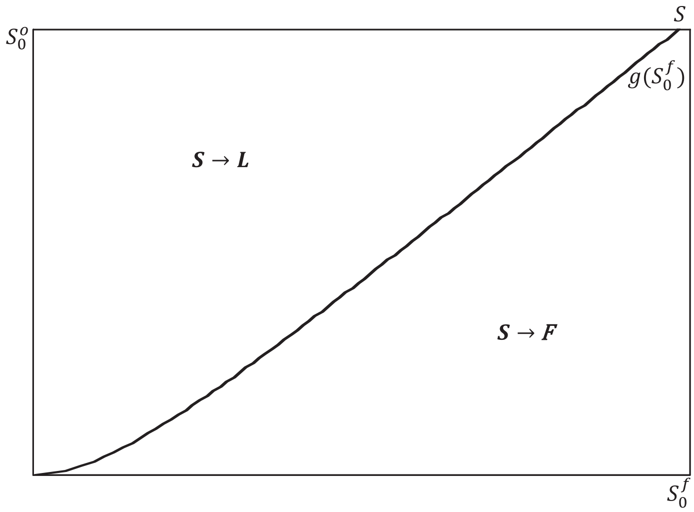

Fig. 1. Equilibrium with strong market power ($n \leq n_{\hat{\Pi}}$).

(ii) *Intermediate market power*: $n_{\hat{\Pi}} < n < n_{\tilde{\Pi}}$ implying $\tilde{\Pi} < 0$ and $\hat{\Pi} > 0$.
(iii) *Weak market power*: $n \geq n_{\tilde{\Pi}}$ implying $\tilde{\Pi} \geq 0$ and $\hat{\Pi} > 0$.

In the next subsections we characterize the equilibrium for each of these cases.

3.1. Strong market power ($n \leq n_{\hat{\Pi}}$)

In this case, there exists a locus in (S_0^o, S_0^f)-space that separates the region where the equilibrium reads $S \to F$ and the region where it reads $S \to L$.[3] Along this locus, the equilibrium reads S. The next lemma characterizes the S-locus for the case at hand.

Lemma 5. *Suppose $n \leq n_{\tilde{\Pi}}$. If the equilibrium reads S , then for any $S_0^f > 0$ there exists a strictly increasing function $g(S_0^f)$ with $g(0) = 0$ and $g(\infty) = \infty$ such that $S_0^o = g(S_0^f)$.*

Proof. See Appendix A.2. □

Fig. 1 graphically depicts the equilibrium for each combination of initial stocks, where the solid line represents the S-locus $g(S_0^f)$. If the initial stock of the oligopolists is large relative to the fringe, the equilibrium reads $S \to L$, whereas the equilibrium reads $S \to F$ if the oligopolists' initial stock is relatively small. A specific intermediate initial stock size (given by the S-locus $g(S_0^f)$) results in simultaneous use throughout. The intuition behind the first outcome directly follows from the explanation below Lemma 4: the oligopolists will choose a limit-pricing strategy as soon as the fringe's stock is depleted (because marginal profits at $p = b$ are non-positive) and they will never supply enough to prevent the fringe from entering the market when its stock is still positive (because marginal profits at $p = k^f$ are negative). Formally,

Proposition 1. *Suppose $n_{\hat{\Pi}} \equiv \frac{\alpha - b}{b - k^o} > 1$ and $n \leq n_{\hat{\Pi}}$. Let $S_0^f > 0$ be given. The equilibrium reads*

(i) *S if and only if $S_0^o = g(S_0^f)$,*
(ii) *$S \to F$ if and only if $S_0^o < g(S_0^f)$,*
(iii) *$S \to L$ if and only if $S_0^o > g(S_0^f)$.*

Proof. See Appendix A.3. □

3.2. Intermediate market power ($n_{\hat{\Pi}} < n < n_{\tilde{\Pi}}$)

In this case, marginal profits of the oligopolists during limit pricing are positive.[4] As a result, whether oligopolists choose a limit-pricing strategy depends on their remaining stock at the time the fringe's stock is depleted. A limit-pricing phase may still occur in equilibrium, but with a maximum duration denoted $\hat{T}$, and with the corresponding limit-pricing phase

[3] Note that this case is relevant only if $n_{\hat{\Pi}} \geq 1$.
[4] Note that this case is relevant only if $n_{\hat{\Pi}} > 1$.

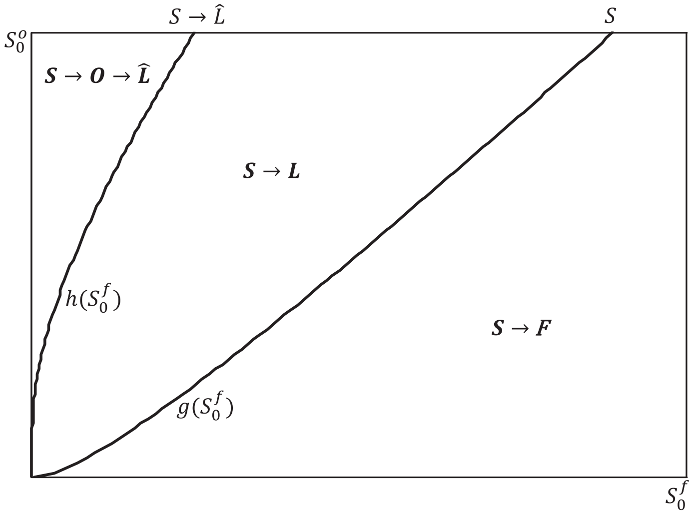

Fig. 2. Equilibrium with intermediate market power ($n_{\hat{\Pi}} < n < n_{\tilde{\Pi}}$).

denoted $\hat{L}$. This maximum duration follows from (3b) evaluated at the start of the limit pricing regime (with $q^f = \mu^o = 0$ imposed) and from (4) evaluated at T_L. Moreover, in this case we have an additional locus, in (S_0^o, S_0^f)-space, at which the equilibrium reads $S \rightarrow \hat{L}$. This locus corresponds to a knife-edge case, and separates the region where the equilibrium reads $S \rightarrow L$ from the region where it reads $S \rightarrow O \rightarrow \hat{L}$. Lemma 6 characterizes the $S \rightarrow \hat{L}$-locus.

Lemma 6. *Suppose* $n_{\tilde{\Pi}} \equiv \frac{\alpha - k^f}{k^f - k^o} > 1$ *and* $n_{\hat{\Pi}} < n < n_{\tilde{\Pi}}$. *Then:*

(i) *If the equilibrium reads S, then for any* $S_0^f > 0$ *there exists a strictly increasing function* $g(S_0^f)$ *with* $g(0) = 0$ *and* $g(\infty) = \infty$ *such that* $S_0^o = g(S_0^f)$.

(ii) *If the equilibrium reads* $S \rightarrow \hat{L}$, *then for any* $S_0^f > 0$ *there exists a strictly increasing function* $h(S_0^f)$ *with* $h(0) > 0$, $h(\infty) = \infty$, *and* $h(S_0^f) > g(S_0^f)$ *such that* $S_0^o = h(S_0^f)$.

Proof. See Appendix A.2. □

Fig. 2 shows the equilibrium for each combination of initial stocks. The lower line in the figure represents the S-locus $g(S_0^f)$ and the upper line depicts the $S \rightarrow \hat{L}$-locus $h(S_0^f)$. Intuitively, since $\hat{\Pi} > 0$, oligopolists do not necessarily engage in limit pricing as soon as the fringe's stock is depleted: It may be optimal for them to supply an amount of the resource that is large enough to drive the price below the price of renewables, implying that an intermediate O phase may occur after the fringe's stock is depleted, provided that the initial stock of the oligopolists is large enough. However, because $\tilde{\Pi} < 0$, before depletion of the fringe's stock the oligopolists will not supply enough to drive the price down to an extent that prevents the fringe from entering the market. Hence, an initial phase O cannot occur. Formally,

Proposition 2. *Suppose* $n_{\tilde{\Pi}} \equiv \frac{\alpha - k^f}{k^f - k^o} > 1$ *and* $n_{\hat{\Pi}} < n \leq n_{\tilde{\Pi}}$. *Let* $S_0^f > 0$ *be given. The equilibrium reads*

(i) *S if and only if* $S_0^o = g(S_0^f)$,
(ii) $S \rightarrow F$ *if and only if* $S_0^o < g(S_0^f)$,
(iii) $S \rightarrow \hat{L}$ *if and only if* $S_0^o = h(S_0^f)$,
(iv) $S \rightarrow L$ *if and only if* $S_0^o \in (g(S_0^f), h(S_0^f))$,
(v) $S \rightarrow O \rightarrow \hat{L}$ *if and only if* $S_0^o > h(S_0^f)$.

Proof. See Appendix A.3. □

3.3. Weak market power ($n > n_{\tilde{\Pi}}$)

Due to the relatively large marginal profits of the oligopolists, in this case an initial O phase may occur before the fringe's stock is depleted: It may be profitable for the oligopolists to drive the price down to such an extent that entry of the fringe is prevented initially. The locus where the equilibrium reads S is now only defined for $S_0^f \in [0, S_M^f]$, where S_M^f is the extremum of the S-locus (it can be seen from (9a) that an extremum exists in this case). Furthermore, as shown in Fig. 3, we have three additional curves that separate the different equilibrium regions. The first additional locus separates the region where

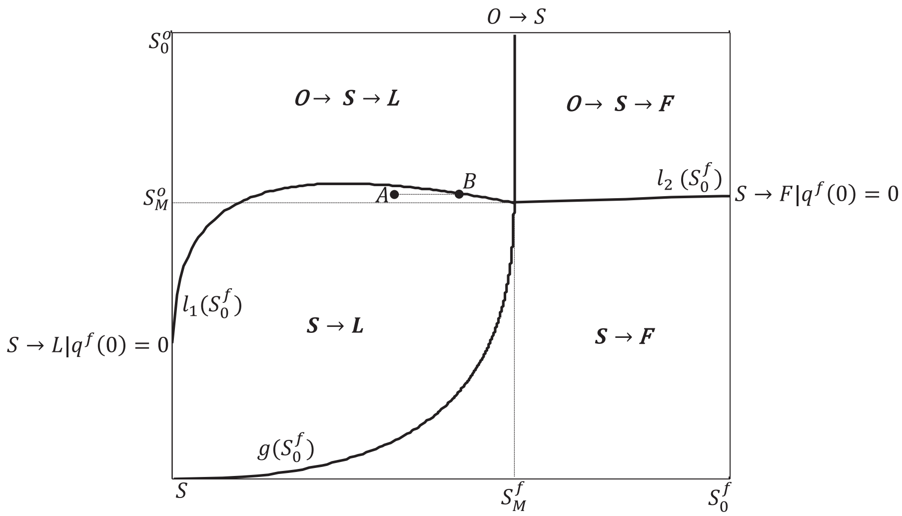

Fig. 3. Equilibrium with weak market power ($n > n_{\tilde{\Pi}}$).

the equilibrium reads $S \to L$, and the region where it reads $O \to S \to L$. The second one separates the $S \to F$ region and the $O \to S \to F$ region. The third additional locus separates the $O \to S \to F$ region from the $O \to S \to L$ region. We will first discuss the properties of the four loci in Lemmata 7–9 and then move on to fully characterizing the equilibrium in Proposition 3.

The next Lemma characterizes the S-locus and the $O \to S$ -locus.

Lemma 7. *Suppose $n > n_{\tilde{\Pi}}$. Define $\tilde{T}^S$ by*

$$e^{r\tilde{T}^S} = 1 + \frac{\alpha - b}{n\tilde{\Pi}}, \tag{13a}$$

and (S_M^o, S_M^f) by

$$r\beta S_M^f = -n\tilde{\Pi}(r\tilde{T}^S - 1 + e^{-r\tilde{T}^S}) + (\alpha - b)(1 - e^{-r\tilde{T}^S}), \tag{13b}$$

$$r\beta S_M^o = n(k^f - k^o)(r\tilde{T}^S - 1 + e^{-r\tilde{T}^S}). \tag{13c}$$

Then it holds that:

(i) *If the equilibrium reads S, for any $S_0^f \in [0, S_M^f]$ there exists a strictly increasing function $g(S_0^f)$ such that $S_0^o = g(S_0^f)$ with $g(0) = 0$. Moreover, $g^{-1}(S_0^o)$ reaches a maximum at point (S_M^o, S_M^f).*

(ii) *If the equilibrium reads $O \to S$, it holds that $S_0^f = S_M^f$ and $S_0^o > S_M^o$.*

Proof. See Appendix A.2. □

For $S_0^f \le S_M^f$, at the locus between the region where the equilibrium sequence reads $S \to L$, and the region where the equilibrium sequence is $O \to S \to L$ the two equilibria should coincide. This implies that $T^O = 0$ and, from (6c) and (7c), that $q^f(0) = 0$. We denote this locus by $S \to L|_{q^f(0)=0}$ in Fig. 3 and characterize it in Lemma 8.

Lemma 8. *Suppose $n > n_{\tilde{\Pi}}$. If the equilibrium reads $S \to L$ with $q^f(0) = 0$, then for any $S_0^f \in [0, S_M^f]$ there exists a function $l_1(S_0^f)$ such that $S_0^o = l_1(S_0^f)$, with the following properties:*

(i) $l_1(0) > 0$,
(ii) $l_1(S_M^f) = S_M^o$,
(iii) $\lim_{S_0^f \to 0} l_1'(S_0^f) = \infty$,
(iv) $l_1'(S_M^f) = -1 + \frac{1}{n}\left(1 - \frac{(k^f - k^o)(1+n)}{b - k^o}\right) + \frac{(k^f - k^o)n}{(k^f - k^o)n - (b - k^f)}$.

Proof. See Appendix A.2. □

In words, properties (i) and (ii) say that the $S \to L|_{q^f(0)=0}$-locus starts above the horizontal axis in the (S_0^f, S_0^o)-plane in Fig. 3 and that it cuts the S-locus at its extremum. Property (iii) implies that it is vertical at the vertical intercept. According to property (iv), the slope of the locus can either be positive or negative near (S_M^o, S_M^f). If the slope is negative (the case shown in Fig. 3), the properties together imply that the locus must be non-monotonic. The consequences of this non-monotonicity will be discussed in Proposition 3.

For $S_0^f > S_M^f$, at the boundary between the region where the equilibrium sequence reads $S \to F$, and the region where the equilibrium sequence is $O \to S \to F$, the two equilibria must coincide, which requires $T^O = q^f(0) = 0$. In Fig. 3 this locus is labeled $S \to F|_{q^f(0)=0}$. Lemma 9 characterizes the $S \to F|_{q^f(0)=0}$-locus.

Lemma 9. *Suppose $n > n_{\tilde{\Pi}}$. If the equilibrium reads $S \to F$ with $q^f(0) = 0$, then for any $S_0^f > S_M^f$ there exists a strictly increasing function $l_2(S_0^f)$ with $l_2(S_M^f) = l_1(S_M^f)$ such that $S_0^o = l_2(S_0^f)$.*

Proof. See Appendix A.2. □

For ease of presentation, to fully characterize the equilibrium in Proposition 3 we split the state space into two regions: a region where $S_0^f \in [0, S_M^f]$ (part (i) of the proposition) and a region where $S_0^f > S_M^f$ (part (ii) of the proposition).

Proposition 3. *Suppose $n > n_{\tilde{\Pi}} \equiv \frac{\alpha - k^f}{k^o - k^f}$. Define S_M^f and S_M^o by (13a)- (13c). Then:*

(i) For all $S_0^f \in [0, S_M^f]$ the equilibrium reads

- (a) *S if and only if $S_0^o = g(S_0^f)$,*
- (b) *$S \to F$ if and only if $S_0^o < g(S_0^f)$,*
- (c) *$S \to L$ if and only if $S_0^o \in (g(S_0^f), l_1(S_0^f)]$,*
- (d) *$O \to S \to L$ if and only if $S_0^o > l_1(S_0^f)$,*
- (e) *$O \to S$ if and only if $S_0^f = S_M^f$ and $S_0^o > S_M^o$.*

(ii) For all $S_0^f > S_M^f$ the equilibrium reads

- (a) *$S \to F$ if and only if $S_0^o \le l_2(S_0^f)$,*
- (b) *$O \to S \to F$ if and only if $S_0^o > l_2(S_0^f)$.*

Proof. See Appendix A.3. □

The non-monotonicity of the $S \to L|_{q^f(0)=0}$-locus can be used to establish the following result.

Corollary 1. *An increase in S_0^f can result in a decrease in the initial extraction of the fringe.*

Proof. Immediate from Lemma 8 and Proposition 3. □

This result can be illustrated using Fig. 3. Consider a point (S_0^o, S_0^f) in the interior of the region $S \to L$ with $S_0^o > S_M^o$, e.g., point A in Fig. 3; then one can increase S_0^f by an amount Δ such that point B with coordinates $(S_0^o, S_0^f + \Delta)$ is just on the $S \to L|_{q^f(0)=0}$-locus. Such an increase in the stock of the fringe alone results in the fringe producing nothing at time 0 by definition of the locus. Moreover, an increase of S_0^f slightly beyond the amount Δ would result in an $O \to S \to L$ equilibrium, implying that the increase in the stock of the fringe now results in a whole interval of time during which the fringe abstains from production.

The intuition behind this result becomes clear when we decompose the effect of an increase in the fringe's initial stock on its initial extraction in the $S \to L$-region into two effects: a *scarcity* effect and a *crowding out* effect. An increase in the fringe's resource stock lowers the initial price and increases total initial extraction, as the resource becomes less scarce. This is the *scarcity effect*, which tends to boost initial extraction by the fringe. However, an increase in the resource stock of the fringe relative to that of the oligopolists also lowers the duration of the final limit pricing regime. As a result, extraction by the oligopolists is front-loaded. This front-loading amplifies the increase in initial extraction by the oligopolists due to the scarcity effect, but reduces extraction by the fringe. If this *crowding out effect* is strong enough, the increase in the fringe's initial stock leads to a reduction in its initial extraction.

The scarcity effect is stronger if the resource stock is small, which explains why the $S \to L|_{q^f(0)=0}$-locus in Fig. 3 is upward-sloping for low values of the fringe's stock. At points on the negatively-sloped part of the locus, the scarcity effect is dominated by the crowding out effect. The locus would be downward-sloping throughout if the crowding out effect would always be dominant and, conversely, it would be upward-sloping if the scarcity effect would be dominant throughout.

Regarding the former case, note that the scarcity effect is eliminated entirely by considering the case where the fringe's extraction costs get arbitrarily close to the marginal production costs of renewables, which would imply a zero scarcity rent for the fringe's stock and therefore a monotonically downward-sloping locus.[5] Regarding the latter case, note that property (iv) of Lemma 8 implies that the slope of the $S \to L|_{q^f(0)=0}$-locus depends positively on the marginal production costs of the

[5] The slope of the $S \to L|_{q^f(0)=0}$-locus is derived in (A.11). From this expression we obtain $\lim_{k^f \to b} l_1'(S_0^f) = -1$.

backstop: $\partial l_1'(S_M^f)/\partial b > 0$. The reason is that an increase in the renewables production costs strengthens the scarcity effect (as discussed above) and weakens the crowding-out effect as the extraction rate during limit pricing will fall. Hence, if the renewables costs exceed a certain threshold, the $S \to L|_{q^f(0)=0}$-locus will be upward-sloping throughout.[6]

4. Extensions

So far, we have assumed that marginal extraction costs are independent of the remaining resource stock and that firms use open-loop strategies. In this section, we will first examine the effect of stock-dependent extraction costs and subsequently discuss the merits and disadvantages of our open-loop equilibrium concept.

4.1. Stock-dependent extraction costs

Instead of constant marginal extraction costs, we now assume that the marginal extraction costs of the fringe increase as its remaining resource stock becomes smaller.[7] Following Salo and Tahvonen (2001), we assume that the fringe's marginal extraction cost function reads $K^f(\tilde{S}^f) = c_0^f - c_1^f \tilde{S}^f$, where $\tilde{S}^f$ denotes the *physical* resource stock, and $c_0^f > 0$ and $c_1^f \geq 0$ are cost parameters. We define $\bar{S}^f$ by $c_0^f - c_1^f \bar{S}^f = b$ and let $S^f \equiv \tilde{S}^f - \bar{S}^f$ denote the *economic* resource stock. The marginal extraction costs can then be written as

$$K^f(S^f) = b - c_1^f S^f. \tag{14}$$

Given that the fringe consists of a large number of atomistic firms, it does not take into account the effect of its current extraction on its future extraction costs: at the individual firm level, extraction is not stock-dependent, as each firm only owns an infinitesimal small amount of the resource. But at the aggregate level, the extraction costs of the fringe depend on its remaining reserve. Accordingly, besides the time path of the price, the fringe also takes the time path of its marginal extraction costs as given. Furthermore, in order to clearly show the effect of the stock-dependency of the fringe's marginal extraction costs, we keep the marginal extraction costs of the oligopolists constant, as in our baseline model. Analytically solving the model with stock-dependent marginal extraction costs requires a different approach from the one we have taken in this paper. Instead, we will only provide a numerical example in order to show the effects of stock-dependent extraction costs. The parameter values that we use are $\alpha = 10$, $\beta = 1$, $k^o = 1$, $b = 5$, and $n = 1$. For the baseline model we use $k^f = 3$, implying that the 'intermediate market power regime' of Section 3.2 applies. For the model with stock-dependent extraction costs we pick $c_1^f = 2/100$, yielding $K^f(100) = 3 = k^f$. Hence, for $S_0^f = 100$ marginal extraction costs are initially the same as in the baseline model, and gradually increase to b over time, at which point the economic stock of the fringe is depleted.

Panel (a) of Fig. 4 shows the extraction paths of the oligopolist (solid) and the fringe (dashed) for the baseline model (black) and the model with stock-dependent extraction costs (grey) for $S_0^f = S_0^o = 100$. The figure shows that the fringe front-loads its extraction because of the increasing marginal extraction costs over time. The oligopolist responds by lowering its initial extraction and increasing it later on. The sequence of extraction does not differ between the equilibria: it is $S \to L$ in both cases.

Panel (b) shows the optimal trajectories in stock-space, for different combinations of the initial resource stocks with $S_0^f \in (0, 100)$ and $S_0^o \in (0, 100)$. Over time, the resource stocks move along the dashed curves in south-western direction, or along the horizontal (vertical) axis towards the origin as soon as the stock of the oligopolist (fringe) is depleted. The black curves correspond to the model with constant and the grey curves to the model with stock-dependent extraction costs. The slope of these curves equals q^o/q^f for a given combination of resource stocks. The figure shows that for relatively high initial resource stocks (i.e., in the right part of the figure), the grey curves are flatter than the black ones, implying that initial extraction by the fringe is relatively higher in the model with stock-dependent extraction costs, as shown in panel (a) as well. However, if the fringe starts with a relatively low initial resource stock (i.e., in the upper left part of the panel (b)), this is reversed. Furthermore, in both models the extraction sequence reads $S \to F$ if the trajectory starts in the lower right part of the figure and S, $S \to L$, $S \to \hat{L}$, or $S \to O \to \hat{L}$ otherwise.

4.2. Equilibrium concept

According to the open-loop equilibrium concept that we use in our paper, firms determine and commit to their extraction path at the outset. In principle, however, extractive firms can condition their actual extraction on the existing fossil fuel stocks, owned by themselves as well as by other suppliers, thereby justifying the use of a closed-loop equilibrium concept. Nevertheless, there are several, and diverse, reasons why it may be appropriate to consider the set of open-loop strategies.

A first justification stems from the prevalence of long term contracts in non-renewable resource markets so that actual extraction rates do not merely depend on actual stocks and a form of precommitment applies. See Liski and Montero (2014) on this.

[6] Lemma 8(iv) implies that $l_1'(S_M^f) > 0$ if $b \in \left(\frac{k^f(n-2)+3k^o n+(k^f-k^o)n\sqrt{5+4n}}{2(n-1)}, \alpha\right]$.

[7] Applied to the oil market, this assumption captures the idea that extraction costs differ between various types of oil that are owned by small producers, such as deep water oil, oil shale and oil sands (cf. Fischer and Salant, 2017).

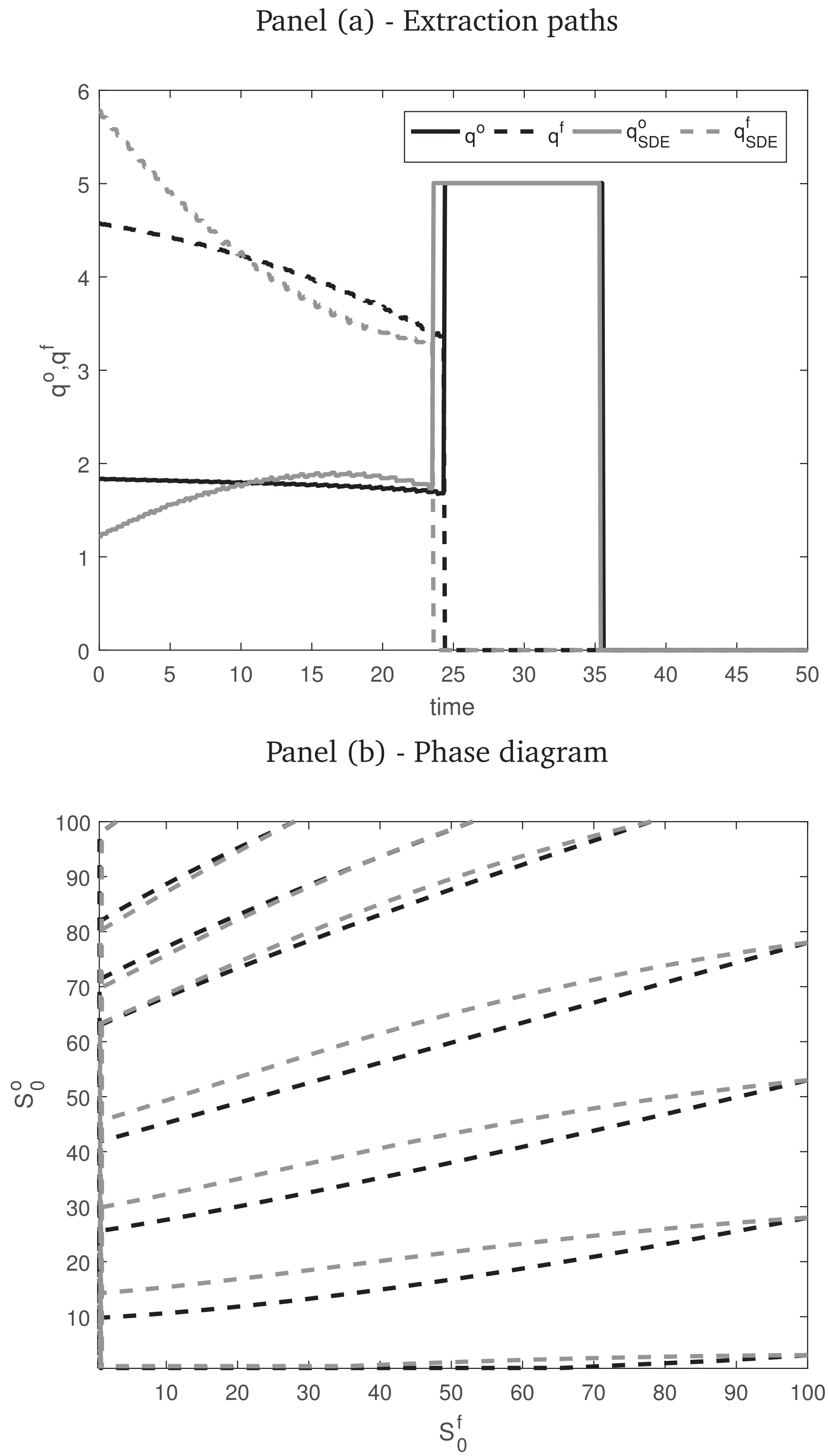

Fig. 4. The effect of stock-dependent extraction costs. *Notes:* Panel (a) shows the extraction paths of the oligopolist (solid) and the fringe (dashed) for the baseline model (black) and the model with stock-dependent extraction costs (grey) for $S_0^f = S_0^o = 100$. *The dashed black (grey) lines in Panel (b) show the optimal trajectories in stock-space for the model with constant (stock-dependent) marginal extraction costs, for different combinations of the initial resource stocks with* $S_0^f \in (0, 100)$ *and* $S_0^o \in (0, 100)$. *The parameter values are* $\alpha = 10,\ \beta = 1,\ k^o = 1,\ k^f = 3,\ b = 5,\ n = 1,$ *and* $c_1^f = 2/100$.

A second justification could be that sizes of existing resource stocks are uncertain. In particular, resource owners may be uncertain about their own stock, but they may also try to keep the size of their own stock secret to other resource owners (cf. Gerlagh and Liski, 2014). Hence there is a case for arguing that resource stocks are at best imperfectly known. While the open-loop equilibrium requires information of the initial vector of stocks, a closed-loop equilibrium requires information on the vector stocks at each moment.

A third justification can be found in the work by Eswaran and Lewis (1985). As far as we know, this is the only paper that compares open-loop and closed-loop equilibria in a non-renewable oligopoly and in a cartel-fringe setting under private property. They conclude that the two outcomes coincide for the cartel-fringe equilibrium. For the oligopoly, open and closed-loop equilibria coincide when demand is iso-elastic and extraction costs are zero, or when the oligopolists are symmetric, extraction costs are quadratic, and demand is linear. Eswaran and Lewis (1985) show numerically that, even if they do not exactly coincide, the open and closed-loop equilibria do not differ substantially under more general conditions (e.g., asymmetric oligopolists and stock-dependent extraction costs). However, their model, unlike ours, does not have a backstop technology nor does it have a price-taking fringe if the number of oligopolists exceeds unity.

To fully account for these features in a closed-loop equilibrium is a challenging task, even when resorting to a numerical approach. In addition to the computational burden of numerically characterizing the closed-loop equilibrium due to the curse of dimensionality, a second difficulty lies in characterizing an equilibrium with limit pricing. While in open-loop it is natural to focus on symmetry under limit pricing because the oligopolists are identical, in a closed-loop equilibrium we have to examine limit pricing without resorting to this important feature. The reason is that in a closed-loop equilibrium we need to specify the strategies for all vectors of stocks, including the cases where firms' stocks are not equal. Characterizing the equilibrium under limit pricing when firms are asymmetric is in itself a problem that involves competition between oligopolists that face a joint production constraint.

In sum, while characterizing the closed-loop equilibrium would be novel and would offer an interesting robustness check to our qualitative results, it is a task that commands a substantial additional amount of investigative work. The only paper that we are aware of that offers an analytical solution to a duopoly in non-renewable resources under private property using closed-loop strategies is Salo and Tahvonen (2001). They consider stock-dependent linear extraction costs and allow for asymmetry. The resource is then asymptotically depleted and the infinite time horizon allows to obtain an analytical solution. However, including a backstop renders the model intractable analytically with the same difficulties that were highlighted above. Moreover, with constant marginal extraction cost, as we assume, their specification of the strategies can be shown not to work.

A complicating factor is also the assumption of the existence of a price-taking fringe in our model. Benchekroun and Withagen (2012) point at this difficulty. One would hope that price-taking would constitute the limit of the number of fringe members going to infinity. Hence, one would start with a finite number of fringe members, each with full knowledge of all existing stocks and each conditioning its actions on these stocks. However, Benchekroun and Withagen (2012) show in the context of a resource model without renewables but with a single monopolist, that this conjecture is invalid, due to the fact that also in the limit the cartel takes into account how its own stock affects the behavior of all fringe members. Hence, the open-loop cartel-fringe model cannot be considered the limit of a closed-loop model and therefore price-taking is a modeling choice from the outset and cannot be rationalized as the limit of an endogenous process. The same problem arises here in our model, of course.

In spite of these considerations we conjecture that many of the qualitative properties of open-loop will go through in closed-loop with identical oligopolists, such as the ordering of the phases in an equilibrium. Investigating this further through numerical exercises would be a promising route for future research.

5. Concluding remarks

We have fully characterized the equilibrium in a framework that captures three key features of the oil market: the presence of players with market power, the presence of small price-taking players and the presence of renewables to which demand will shift if the oil price increases beyond a certain threshold. An interesting theoretical implication of the equilibrium is the possibility that an increase in the fringe's stock results in a decrease in its initial supply. This is to the best of our knowledge a novel result in the theory of resource economics.

Furthermore, we have shown that the sequence of extraction phases in equilibrium crucially depends on the oligopolists' market power. In case of strong or intermediate market power, the equilibrium starts with a phase of simultaneous supply by the oligopolists and the fringe. In case of weak market power, however, there may be an initial phase during which the oligopolists are the sole suppliers. We have also fully characterized the conditions under which oligopolists perform a limit pricing strategy by supplying just enough oil to drive the oil price marginally below the production costs of renewable producers. In case of strong market power, limit pricing will occur as soon as the fringe's stock is depleted. In case of intermediate or weak market power, however, limit pricing will only occur after the fringe's stock is depleted and the oligopolists' remaining stocks are below a certain threshold.

Several well-studied market structures can be retrieved as special cases of our framework: in particular the situations where fossil fuels are supplied under perfect competition, monopoly, or in a cartel-fringe setting. This has the benefit of better understanding and qualifying some of the strong conclusions of each of the polar models. Indeed, the full characterization of the equilibrium within our framework can be a useful tool to obtain insight in the implications of different policy instruments such as a renewables subsidy or a carbon tax. For example, under perfect competition, a subsidy for renewables results, in equilibrium, in an increase of the industry's initial extraction.[8] The intuition behind this unintended consequence of a subsidy is that by making renewables cheaper, the subsidy lowers the future market price of oil. Consequently, resource owners deplete their stocks more rapidly. In contrast, when the resource is supplied by a monopoly, a renewables subsidy induces a decrease in the initial extraction (as long as the monopoly does not adopt limit pricing from the beginning). In so doing, the monopolist is able to postpone entry of renewables producers (cf. Gilbert and Goldman, 1978; Hoel, 1983; Van der Meijden and Withagen, 2016). In our framework it can be shown that the effect of an increase in the renewables subsidy on initial extraction is ambiguous and depends on the stocks of the cartel and the fringe. As long as the initial aggregate stock of the cartel is small relative to that of the fringe, the perfectly competitive mechanism can be shown to dominate, implying that the industry's initial extraction goes up in response to an increase in the renewables subsidy. However, if marginal profits during limit pricing are positive and if the initial aggregate stock of the cartel is relatively large, the monopolistic mechanism can be shown to dominate and initial extraction decreases as a result of an increase in the renewables subsidy.

Natural and relevant applications of the results of this paper include examining the optimal choice of a tax on carbon or the optimal subsidy to renewables. Also, (numerically) examining the properties of the closed-loop equilibrium would be interesting. These extensions are left for future research.

Appendix A

In this appendix, we first characterize the initial stocks and phase switching times for all equilibrium candidates in Lemmata A.1–A.9. We then prove Lemmata 3–9. Finally, we prove Proposition 1, 2 and 3.

A.1. Necessary conditions for each equilibrium candidate

In this section, we identify the necessary conditions for all equilibrium candidates in Lemmata A.1–A.9 (apart from the equilibrium candidate consisting of S throughout, for which the necessary conditions are already derived in Lemma 2). To establish these conditions we use the extraction paths found under each regime in Lemma 1, which, combined with the resource constraints, the transversality condition and price continuity at each transition in an equilibrium sequence, gives a set of conditions that the vector of initial stocks and the terminal dates of each regime in an equilibrium sequence must satisfy.

Lemma A.1. *Given (S_0^f, S_0^o), if the equilibrium reads $S \to F$, then there exist T^S and T^F with $0 < T^S < T^F$ such that*

$$r\beta S_0^f = -n(k^f - k^o)(rT^S - 1 + e^{-rT^S}) + (b - k^f)(rT^F - 1 + e^{-rT^F}) + (\alpha - b)rT^F, \tag{A.1a}$$

$$r\beta S_0^o = n(k^f - k^o)(rT^S - 1 + e^{-rT^S}). \tag{A.1b}$$

Proof. For $t \in S$ we have (6d) and (6e). For $t \in F$ we have (5d). Furthermore, price continuity at the final time T^F and (5b) imply $\lambda^f = (b - k^f)e^{-rT^F}$. Also, price continuity at the transition time T^S together with (5c), (6b) and (6c) give $\lambda^o = (k^f - k^o)e^{-rT^S} + \lambda^f = (k^f - k^o)e^{-rT^S} + (b - k^f)e^{-rT^F}$. Solving the integrals $\int_0^{T^F} q^f dt = S_0^f$ and $\int_0^{T^S} q^o dt = S_0^o$ yields the result. □

Lemma A.2. *Given (S_0^f, S_0^o), if the equilibrium reads $S \to L$, then there exist T^S and T^L with $0 < T^S < T^L$ such that*

$$r\beta S_0^f = -n\tilde{\Pi}(rT^S - 1 + e^{-rT^S}) + [\alpha - b](1 - e^{-rT^S}) + n(b - k^o)(e^{rT^S - rT^L} - 1)(1 - e^{-rT^S}), \tag{A.2a}$$

$$r\beta S_0^o = n(k^f - k^o)(rT^S - 1 + e^{-rT^S}) + n(1 - e^{-rT^S})(1 - e^{rT^S - rT^L})(b - k^o) + (\alpha - b)(rT^L - rT^S). \tag{A.2b}$$

Proof. For $t \in S$ we have (6d) and (6e). For $t \in L$ phase we have (8d). From price continuity at the final time T^L together with (4) we obtain $\lambda^o = (b - k^o)e^{-r\tilde{T}^L L}$. It follows from price continuity at the transition time T_S and from (6b) that $\lambda^f = (b - k^f)e^{-rT^S}$. Solving the integrals $\int_0^{T^S} q^f dt = S_0^f$ and $\int_0^{T^L} q^o dt = S_0^o$ yields the result. □

Lemma A.3. *Given (S_0^f, S_0^o), if the equilibrium reads $S \to O \to \hat{L}$, then there exist T^S, T^O and T^L with $0 < T^S < T^O < T^L$ such that*

$$r\beta S_0^f = -n\tilde{\Pi}(rT^S - 1 + e^{-rT^S}), \tag{A.3a}$$

$$r\beta S_0^o = \frac{n^2}{n+1}\tilde{\Pi}(rT^S - 1 + e^{-rT^S}) + \frac{n}{n+1}\hat{\Pi}(rT^O - 1 + e^{-rT^O}) + (\alpha - b)rT^L, \tag{A.3b}$$

$$(b - k^o)e^{-rT^L} = \hat{\Pi}e^{-rT^O}. \tag{A.3c}$$

[8] This unintended outcome of a green policy is known as the Green Paradox (cf. Sinn, 2008; 2012).

Proof. For $t \in S$ we have (6d) and (6e). For $t \in O$ we have (7d). For $t \in \hat{L}$ we have (8d). From price continuity at the final time T^L together with (4) we obtain $\lambda^o = (b-k^o)e^{-rT^L}$. Furthermore, price continuity at T^S together with (6b), (7c) and (7d) imply $(n+1)(k^f + \lambda^f e^{rT^S} = (\alpha + n(k^o + \lambda^o e^{rT^S}))$. Condition (A.3c) is obtained by combining (6c), (8d), and (4) and using price continuity at T^O. Solving the integrals $\int_0^{T^S} q^f dt = S_0^f$ and $\int_0^{T^L} q^o dt = S_0^o$ yields the result. □

Lemma A.4. *Given S_0^f, if the equilibrium reads $S \to \hat{L}$, then there exist T^S, T^L and S_0^o with $0 < T^S < T^L$ such that*

$$r\beta S_0^f = -n\tilde{\Pi}(rT^S - 1 + e^{-rT^S}), \tag{A.4a}$$

$$r\beta S_0^o = \frac{n}{n+1}\left(n\tilde{\Pi} + \hat{\Pi}\right)(rT^S - 1 + e^{-rT^S}) + (\alpha - b)rT^L, \tag{A.4b}$$

$$(b-k^o)e^{-rT^L} = \hat{\Pi}e^{-rT^S}. \tag{A.4c}$$

Proof. For $t \in S$ we have (6d) and (6e). For $t \in \hat{L}$ we have (8d). From price continuity at the final time T^L together with (4) we obtain $\lambda^o = (b - \tau - k^o)e^{-rT^L}$. Furthermore, price continuity at T^S together with (6b) gives $\lambda^f = (b-k^f)e^{-rT^S}$. Condition (A.4c) is obtained by combining (6c), (8d), and (4) and using price continuity at T^S. Solving the integrals $\int_0^{T^S} q^f dt = S_0^f$ and $\int_0^{T^L} q^o dt = S_0^o$ yields the result. □

Lemma A.5. *Given (S_0^f, S_0^o), if the equilibrium reads $O \to S \to L$, then there exist T^O, T^S and T^L with $0 < T^O < T^S < T^L$ such that*

$$r\beta S_0^f = -n\tilde{\Pi}\left(rT^S - rT^O + 1 - e^{rT^S - rT^O}\right), \tag{A.5a}$$

$$\begin{aligned} r\beta S_0^o = {} & \frac{n}{n+1}(\alpha - k^o)rT^O - \frac{n}{n+1}(b-k^o)e^{-rT^L}(e^{rT^O} - 1) - n((b-k^o)e^{-rT^L} - (b-k^f)e^{-rT^S})(e^{rT^S} - e^{rT^O}) \\ & + n\left[(k^f - k^o)(rT^S - rT^O)\right] + (\alpha - b)(rT^L - rT^S), \end{aligned} \tag{A.5b}$$

$$-n\tilde{\Pi} = ((n+1)(b-k^f)e^{-rT^S} - n(b-k^o)e^{-rT^L})e^{rT^O}. \tag{A.5c}$$

Proof. For $t \in O$ we have (7d). For $t \in S$ we have (6d) and (6e). For $t \in L$ we have (8d). From price continuity at the final time T^L together with (4) we obtain $\lambda^o = (b-k^o)e^{-rT^L}$. Furthermore, price continuity at T^S together with (6b), (7c) and (7d) imply $(n+1)(k^f + \lambda^f e^{rT^S} = (\alpha + n(k^o + \lambda^o e^{rT^S}))$. Condition (A.5c) is obtained by using price continuity at T^O together with (6b), (7c) and (7d). Solving the integrals $\int_{T^O}^{T^S} q^f dt = S_0^f$ and $\int_0^{T^L} q^o dt = S_0^o$ yields the result. □

Lemma A.6. *Given (S_0^f, S_0^o), if the equilibrium reads $O \to S \to F$, then there exist T^O, T^S and T^F with $0 < T^O < T^S < T^F$ such that*

$$r\beta S_0^f = -n\tilde{\Pi}\{rT^S - rT^O + 1 - e^{rT^S - rT^O}\} + (b-k^f)(-1 + e^{rT^S - rT^F}) + (\alpha - k^f)(rT^F - rT^S) \tag{A.6a}$$

$$r\beta S_0^o = \frac{n}{n+1}(\alpha - k^o)rT^O + n(k^f - k^o)\{rT^S - rT^O - 1 + e^{rT^O - rT^S}\} - \frac{n}{n+1}\{(k^f - k^o)e^{-rT^S} + (b-k^f)e^{-rT^F}\}(e^{rT^O} - 1), \tag{A.6b}$$

$$-n\tilde{\Pi} = (b-k^f)e^{rT^O - rT^F} - n(k^f - k^o)e^{rT^O - rT^S}. \tag{A.6c}$$

Proof. For $t \in O$ we have (7d). For $t \in S$ we have (6d) and (6e). For $t \in F$ we have (5d). Price continuity at the final time T^F together with (5b) gives $\lambda^f = (b-k^f)e^{-rT^F}$. Furthermore, price continuity at T^S together with (6b), (7c) and (7d) imply $(n+1)(k^f + \lambda^f e^{rT^S} = (\alpha + n(k^o + \lambda^o e^{rT^S}))$. Condition (A.6c) is obtained by using price continuity at T^O together with (6b), (7c) and (7d). Solving the integrals $\int_{T^O}^{T^F} q^f dt = S_0^f$ and $\int_0^{T^S} q^o dt = S_0^o$ yields the result. □

Lemma A.7. *Given S_0^o, if the equilibrium reads $O \to S$, then $S_0^f = S_M^f$ and there exist T^O and T^S with $0 < T^O < T^S$ such that*

$$r\beta S_0^f = -n\tilde{\Pi}\{r(T^S - T^O) + 1 - e^{rT^S - rT^O}\}, \tag{A.7a}$$

$$r\beta S_0^o = \frac{n}{n+1}(\alpha - k^o)rT^O - \frac{n}{n+1}(b-k^o)e^{-rT^S}(e^{rT^O} - 1), +n(k^f - k^o)\left(rT^S - rT^O - 1 + e^{rT^O - rT^S}\right), \tag{A.7b}$$

$$n\tilde{\Pi} = \left(n\tilde{\Pi} + \alpha - b\right)e^{rT^O - rT^S}. \tag{A.7c}$$

Proof. For $t \in O$ we have (7d). For $t \in S$ we have (6d) and (6e). From price continuity at the final time T^S together with (4) we obtain $\lambda^o = (b - \tau - k^o)e^{-rT^S}$. Equation (A.7c) is obtained from price continuity at T^O together with (6b), (7c) and (7d). Solving the integrals $\int_{T^O}^{T^S} q^f dt = S_0^f$ and $\int_0^{T^S} q^o dt = S_0^o$ yields (A.7a)-(A.7b). since $T^O - T^S$ must satisfy both (A.7a) and (A.7c), there is a unique value of S_0^f such that both conditions hold. Combining (A.7a) and (A.7c) with (13a) and (13b), it follows that this value equals S_M^f as defined in Lemma 7. □

Lemma A.8. *Given* (S_0^f, S_0^o)*, if the equilibrium reads* $S \to L$ *with* $q^f(0) = 0$*, then there exist* T^S *and* T^L S_0^c *with* $0 < T^S < T^L$ *such that*

$$r\beta S_0^f = (\alpha + nk^o - (n+1)k^f)(rT^S + 1 - e^{rT^S}), \tag{A.8a}$$

$$r\beta S_0^o = -(\alpha + nk^o - (n+1)k^f)(rT^S + 1 - e^{rT^S}) + (\alpha - b)rT^L + (b - k^f)(rT^S - 1 + e^{-rT^S}), \tag{A.8b}$$

$$-n\tilde{\Pi} = (n+1)(b - k^f)e^{-rT^S} - +n(b - k^o)e^{-rT^L}. \tag{A.8c}$$

Proof. For $t \in S$ we have (6d) and (6e). For $t \in \hat{L}$ we have (8d). From price continuity at the final time T^L together with (4) we obtain $\lambda^o = (b - \tau - k^o)e^{-rT^L}$. Furthermore, price continuity at T^S together with (6b), (7c) and (7d) imply $(n+1)(k^f + \lambda^f e^{rT^S} = (\alpha + n(k^o + \lambda^o e^{rT^S}))$. Condition (A.8c) is obtained by imposing $q^f(0) = 0$ in (6d). Combining these expressions and solving the integrals $\int_0^{T^S} q^f dt = S_0^f$ and $\int_0^{T^L} q^o dt = S_0^o$ yields the result. □

Lemma A.9. *Given* (S_0^f, S_0^o)*, if the equilibrium reads* $S \to F$ *with* $q^f(0) = 0$*, then there exist* T^S *and* T^F *with* $0 < T^S < T^F$ *such that*

$$r\beta S_0^f = -n\tilde{\Pi}\{rT^S + 1 - e^{rT^S}\} + (b - k^f)(-1 + e^{rT^S - rT^F}) + (\alpha - k^f)(rT^F - rT^S) \tag{A.9a}$$

$$r\beta S_0^o = n(k^f - k^o)\{rT^S - 1 + e^{-rT^S}\} \tag{A.9b}$$

$$-n\tilde{\Pi} = (b - k^f)e^{-rT^F} - n(k^f - k^o)e^{-rT^S}. \tag{A.9c}$$

Proof. For $t \in S$ we have (6d) and (6e). For $t \in F$ we have (5d). Price continuity at the final time T^F together with (5b) gives $\lambda^f = (b - k^f)e^{-rT^F}$. Furthermore, price continuity at T^S together with (6b), (7c) and (7d) imply $(n+1)(k^f + \lambda^f e^{rT^S} = (\alpha + n(k^o + \lambda^o e^{rT^S}))$.Condition (A.6c) is obtained by imposing $q^f(0) = 0$ in (6d). Solving the integrals $\int_0^{T^F} q^f dt = S_0^f$ and $\int_0^{T^S} q^o dt = S_0^o$ yields the result. □

A.2. Proofs of Lemmata

The proofs of Lemmata 6–9 make use of the necessary conditions derived in Appendix A.1.

Proof of Lemma 3. (i) Suppose $t_1 \in S$ and there exist $t_3 > t_2 > t_1$, such that for all $t_3 > t > t_2$ we have $t \in S$ or $t \in F$. Then $q^f(t) > 0$, $p(t) = k^f + \lambda^f e^{rt}$, $p(t) \le b$ and $\dot{p}(t) > 0$ for all $t_3 > t > t_2$. Therefore $p(t) < b$. But then it is better for the fringe to have $q^f(t_1) > 0$. This is a contradiction. If there would be an O phase after t_1, we obtain a contradiction as well: At t_1, equation (8c) holds. It holds *a fortiori* for all $t > t_1$. Substitution in (7e) gives $p(t) > b$ for all $t \in O$, which is not allowed.

(ii) We have $k^f + \lambda^f e^{rt_1} \le k^o + \lambda^o e^{rt_1}$ from (5b) to (5c). Hence, since $k^f > k^o$, $k^f + \lambda^f e^{rt} < k^o + \lambda^o e^{rt}$ for all $t > t_1$. It follows immediately from (6c) to (7c) that F cannot be followed by S nor by F. Hence, the only transition possible is a direct one to the final phase L. But this is excluded, because it would require an upward jump in the price from (4) to (5b)–(5c).

(iii) Suppose the final regime is O. At the moment of exhaustion of the oligopolists, T^O, the price equals $p(T^O) = b$, because the price is continuous and renewables take over. Also $\beta q^o(T^O) = \alpha - b$. But, from (4) we have $p(T^O) = k^o + \lambda^o e^{rT^O}$, so that from (7d) $\beta q^o(T^O) = \frac{n}{n+1}(\alpha - b)$, contradicting $n < \infty$.

(iv) For $t \in O$ (7a) and (7e) hold. Similarly, for $t \in F$ (5b) and (5c) hold. At the transition time T we then have

$$\begin{aligned} p(T) &= k^f + \lambda^f e^{rT} = \frac{1}{n+1}(\alpha + n(k^o + \lambda^o e^{rT})) \\ &\ge \frac{1}{n+1}(\alpha + n(k^f + \lambda^f e^{rT})) = \frac{1}{n+1}(\alpha + np(T)), \end{aligned}$$

implying $p(T) > \alpha$, a contradiction, because there must be demand at T. □

Proof of Lemma 4. (i) Suppose $\tilde{\Pi} \le 0$ and the initial regime is O. Then it follows from (7a), (7a) and (7b) that $\alpha + nk^o - (n+1)k^f \le ((n+1)\lambda^f - n\lambda^o)e^{rt}$ for all t in the initial interval of time. No transition to F is possible (Lemma 3). A transition to L would imply that the fringe is not producing at all, since L can only be a final regime (Lemma 3). Hence there must be a transition to S, say at T. So, $\alpha + nk^o - (n+1)k^f \le ((n+1)\lambda^f - n\lambda^o)e^{rT}$ because $q^f(T+) \ge 0$ (see (6b)). Since $\alpha + nk^o - (n+1)k^f > 0$ by assumption and O starts before S, we have $(n+1)\lambda^f - n\lambda^o > 0$, so that $((n+1)\lambda^f - n\lambda^o)e^{rt}$ is increasing over time, yielding a contradiction.

(ii) Suppose $\tilde{\Pi} > 0$. For $t \in S$ we have

$$q^f(t) = \frac{1}{\beta}\left(\alpha - (n+1)(k^f + \lambda^f e^{rt}) + n(k^o + \lambda^o e^{rt})\right) \geq 0.$$

Since $\tilde{\Pi} > 0$ it follows that $(n\lambda^o - (n+1)\lambda^f)e^{rt} > 0$ and $q^f(t)$ is increasing over time. In O we have

$$p(t) = \frac{1}{1+n}\{n(k^o + \lambda^o e^{rt}) + \alpha\}.$$

If there would be a transition to S to O price continuity requires

$$p(T^S) = \frac{1}{1+n}\{n(k^o + \lambda^o e^{rt}) + \alpha\} = k^f + \lambda^f e^{rT^S},$$

implying that $\lim_{t\uparrow T^S} q^f(t) = 0$, contradicting that q^f is increasing in S. This proves part (i).

(iii) If $\hat{\Pi} \leq 0$ then $-(\alpha + nk^o - (n+1)b) < e^{rt}\lambda^o$ for all $t \geq 0$. Using this in (8d) and subsequently in (8a) we find $p(t) > b$ for $t \in O$, which is a contradiction. □

Proof of Lemma 5. We define $g(S_0^f)$ as the solution S_0^o of (9a)–(9b). When $\tilde{\Pi} \leq 0$, the right hand side of (9a) is a continuous strictly increasing function of T^S, equal to 0 when $T^S = 0$ and tending to ∞ when $T^S \to \infty$. □

Proof of Lemma 6. Part (i) is similar to the proof of Lemma 5. To prove part (ii), define $h(S_0^f)$ as the solution S_0^o of (A.4a)–(A.4b). For $S_0^f = 0$ and $S_0^o > 0$ we have a positive T^S from (A.4c) if $\hat{\Pi} > 0$. Moreover, h is increasing in T^S and therefore in S_0^f. Also $h(0) > 0$ and $h(\infty) = \infty$. for a given positive S_0^f the end of the simultaneous phase occurs later in the $S \to \hat{L}$ than in the S equilibrium (compare (9a) and (A.4a)). Therefore, the oligopolists' stock required for this regime is higher, stated differently $h(S_0^f) > g(S_0^f)$. □

Proof of Lemma 7. (i) We have defined $g(S_0^f)$ as the solution S_0^o of (9a)–(9b), see Lemma 2. It follows from (13b) to (13c) that

$$r\beta dS_0^f = -n\tilde{\Pi}(r - re^{-r\tilde{T}^S})d\tilde{T}^S + (\alpha - b)re^{-r\tilde{T}^S}d\tilde{T}^S, \tag{A.10a}$$

$$r\beta dS_0^o = n(k^f - k^o)(r - re^{-r\tilde{T}^S})d\tilde{T}^S. \tag{A.10b}$$

Hence, we get $n(k^f - k^o)(1 - e^{-r\tilde{T}^S})d\tilde{T}^S = \beta dS_0^o$, which gives

$$\frac{dS_0^f}{dS_0^o} = \frac{-n\tilde{\Pi}(1 - e^{-r\tilde{T}^S}) + (\alpha - b)e^{-r\tilde{T}^S}}{n(k^f - k^o)(1 - e^{-r\tilde{T}^S})} = \frac{-\tilde{\Pi}}{k^f - k^o} + \frac{\alpha - b}{n(k^f - k^o)(e^{r\tilde{T}^S} - 1)}.$$

It follows that $\frac{dS_0^f}{dS_0^o} \to \infty$ as $\tilde{T}^S \downarrow 0$ and $\frac{dS_0^f}{dS_0^o} < 0$ for $\tilde{T}^S$ sufficiently large (since $\tilde{\Pi} > 0$). Moreover, $\frac{dS_0^f}{dS_0^o}$ is decreasing in $\tilde{T}^S$. Hence $\frac{dS_0^f}{dS_0^o} = 0$ for the unique $\tilde{T}^S$ satisfying (13a).

(ii) If the equilibrium reads $O \to S$ (A.7a)–(A.7c) must hold. We know from Lemma A.7 that this requires $S_0^f = S_M^f$. Furthermore, by imposing $S_0^o = S_M^o$ we have $T^O = 0$ because the S-locus reaches its maximum at (S_M^o, S_M^f). Finally, as $T^S - T^O$ is fixed by (A.7a) and (A.7c) for a given S_0^f, (A.7b) yields $dT^O/dS_0^o > 0$, implying that the duration of the O phase is non-negative for $S_0^o > S_M^o$. □

Proof of Lemma 8. We define $l_1(S_0^f)$ as the solution S_0^o to (A.8a)–(A.8c). For S_0^f we get from (A.8a) that $T^S = 0$, which from (A.8b) implies property (i). Property (ii) is obtained by imposing $T^L = T^S = \tilde{T}^S$, as the S reaches its maximum at (S_M^o, S_M^f), and using the definitions of S_M^f and S_M^o in (13b)–(13c).

Total differentiation of (A.8a)–(A.8c) S_0^f yields

$$l_1'(S_0^f) = \frac{e^{-r(T^L - T^S)}(1+n)(\alpha - b)(b - k^f)}{(e^{rT^S} - 1)\tilde{\Pi}(b - k^o)} + \frac{(1 - e^{-rT^S})n(b - k^f - e^{rT^S}n\tilde{\Pi})}{(e^{rT^S} - 1)\tilde{\Pi}}. \tag{A.11}$$

By noting that $S_0^f \to 0$ implies $T^S \to 0$, we obtain (iii) from taking the limit of this expression for $T^S \to 0$: $\lim_{T^S \to 0} l_1'(S_0^f) = \infty$.

To prove (iv), note again that at (S_M^o, S_M^f) the S-locus reaches its maximum, implying that $T^L = T^S = \tilde{T}^S$. Substitution of this equality together with (13a) into (A.11) yields the expression. □

Proof of Lemma 9. Define $l_2(S_0^f)$ by the solution S_0^o to (A.9a)–(A.9c). By imposing $T^F = T^S$, (A.9c) implies $T^S = \tilde{T}^S$ from (13a) and $S_0^f = S_M^f$ and $S_0^c = S_M^c$ from (13b) to (13c). This proves $l_2(S_M^f) = l_1(S_M^f)$.

Next, we fully differentiate (A.9a)–(A.9c) to get

$$\frac{dS_0^f}{dS_0^o} = \frac{e^{-rT^F} rn\tilde{\Pi}\left[e^{rT^F}(\alpha - k^f - \tau) - (b - k^f)\right]}{(1 - e^{-rT^S})(k^f - k^o)nr(b - k^f)} > 0,$$

where the inequality follows from noting that the part between brackets is larger than $\alpha - (\beta) > 0$. This proves that $l_2(S_0^f)$ is strictly increasing for any $S_0^f > S_M^f$.

A.3. Proofs of propositions

The proofs of Propositions 1, 2 and 3 make use of the necessary conditions derived in Appendix A.1.

Proof of Proposition 1. Using Lemmata 3–4 the only potential sequences are S, $S \to F$ and $S \to L$.

(i) S implies $S_0^o = g(S_0^f)$ from Lemma 5. Suppose then that $S_0^o = g(S_0^f)$ but the equilibrium is not S. If the equilibrium would read $S \to F$ then T^S following from (9b) would have to be equal to the T^F for $S \to F$. This is so because with the given initial stock of the oligopolists we need the same final time. This yields a contradiction, because for the given S_0^o the T^S of (A.1b) would have to coincide with the T^S in (9b). Suppose the equilibrium would be $S \to L$. Then the $S-$phase must last shorter than in the pure S equilibrium. Hence T^S in $S \to L$ is smaller than T^S in S. The initial price should therefore be higher, implying a higher λ^f. But total extraction by the oligopolists in the $S-$phase must be smaller, so that also λ^o is higher (see (6e)). Since the final price is b in both cases, (4) says that the final instant of time T^L in $S \to L$ must be smaller than T^S in S, a contradiction. Hence $S_0^o = g(S_0^f)$ implies S.

(ii) We first show that $S \to F$ implies $S_0^o < g(S_0^f)$. Suppose $S_0^o > g(S_0^f)$ and the equilibrium reads $S \to F$. For the given S_0^o the transition time T^S is uniquely determined by (A.1b). Hence, from (A.1a) the duration of the $F-$phase is a strictly increasing function of S_0^f. Taking $S_0^o > g(S_0^f)$ fixed we increase S_0^f and thereby T^F. But once we reach the manifold where the equilibrium is purely S the length of the $F-$regime collapses to zero, a contradiction. Next, we prove that $S_0^o < g(S_0^f)$ implies $S \to F$. To this end suppose $S_0^o < g(S_0^f)$ and the equilibrium is not $S \to F$. Clearly, we cannot have pure S in this case. Hence, we have $S \to L$ with $T^L > T^S$. If we fix S_0^o and increase S_0^f, (A.2a) implies that T^S must go up. However, (A.2b) then requires that S_0^o increases, which gives a contradiction.

(iii) We first show that $S \to L$ implies $S_0^o > g(S_0^f)$. Suppose $S_0^o < g(S_0^f)$ and the equilibrium reads $S \to L$. The proof follows the proof of the second part of (ii). It remains to be shown that $S_0^o > g(S_0^f)$ implies $S \to L$ as the equilibrium. If this would not hold, the equilibrium is not S. So the equilibrium must be $S \to F$. But it has been shown above that then $S_0^o < g(S_0^f)$, a contradiction. □

Proof of Proposition 2. Because in this case we have $\hat{\Pi} > 0$, an O phase can no longer be ruled out and there exists an upper bound on the length of the duration of the L phase. However since $\tilde{\Pi} < 0$, from Lemma 4, we can rule out having an equilibrium that starts with an O phase as well as a transition from O to S.

The proofs of (i) and (ii) are equal to the proofs for the case $\hat{\Pi} \leq 0$ treated in Proposition 1.

(iii) $S \to \hat{L}$ implies $S_0^o = h(S_0^f)$ from Lemma 6. Suppose then that $S_0^o = h(S_0^f)$ and the equilibrium does not read $S \to \hat{L}$. If the equilibrium would be S or $S \to F$ then the $S-$phase would be shorter than in the case of $S \to \hat{L}$ (see the proof of Lemma 6). This implies a higher λ^f, but also a higher λ^o because somewhere in the $S-$phase there must be a smaller supply from the oligopolists. But that implies from (4) an earlier final instant of time. Hence, not all of the resource is extracted. If the equilibrium would be $S \to O \to \hat{L}$, it can be seen from (A.3a) to (A.3c) that the transition to O occurs at the same time as the transition to $\hat{L}$. Moreover, the lengths of the limit pricing phases are equal. Therefore $S \to O \to \hat{L}$ cannot be an equilibrium.

(iv) Suppose we have $S \to L$ and $S_0^o < g(S_0^f)$. This has been excluded in the Proposition 1. Suppose we have $S \to L$ and $S_0^o > h(S_0^f)$. Keep S_0^o fixed and let S_0^f increase up to the point $\hat{S}_0^f$ where $S_0^o = h(\hat{S}_0^f)$. At that point we have

$$r\beta\hat{S}_0^f = -n\tilde{\Pi}(rT^S - 1 + e^{-rT^S}).$$

In the limit as $S_0^f \to \hat{S}_0^f$

$$e^{rT^S - rT^L} = \frac{\hat{\Pi}}{b - k^o}.$$

If this is inserted in the expression for S_0^f in the $S \to L$ equilibrium we get

$$r\beta\hat{S}_0^f = -n\tilde{\Pi}(rT^S - 1 + e^{-rT^S}) + (1 - e^{-rT^S})(\alpha - b).$$

This yields a contradiction. We next prove that $S_0^o \in (g(S_0^f), h(S_0^f))$ implies $S \to L$. Clearly, the equilibrium is not S or $S \to F$ or $S \to \hat{L}$. So, we only need to exclude $S \to O \to \hat{L}$. We can then repeat the argument used in (iii): If the equilibrium would be $S \to O \to \hat{L}$ then it can be shown that the transition to O occurs at the same time as the transition to $\hat{L}$. Moreover, the lengths of the limit pricing phases are equal. Therefore $S \to O \to \hat{L}$ cannot be an equilibrium.

(v) The proof is straightforward and follows from the necessary and sufficient conditions discussed before. □

Proof of Proposition 3. Using Lemmata 3–4 the only potential sequences are S, $S \to F$, $S \to L$, $O \to S \to F$ and $O \to S \to L$. By noting that the maximum of the S-locus is located at (S_M^f, S_M^o), (a) and (b) of part (i) follow from Proposition 1.

If $S_0^f < S_M^f$ the equilibrium cannot be $O \to S \to F$, because this would require $T^F < T^S$ from (A.6a) to (A.6c). Hence, if $S_0^o > g(S_0^f)$ and $S_0^f > S_M^f$ the only possibilities are $S \to L$ and $O \to S \to L$. At the boundary $S_0^f = l_1(S_0^f)$, (A.2a) to (A.2b) imply $q^f(0) = 0$ and (A.5a) to (A.5c) imply $T^O = 0$. Starting at this boundary, if we increase S_0^o, (A.2a) to (A.2b) would require $q^f(0) < 0$, so that $S \to L$ cannot occur if $S_0^o > l_1(S_0^f)$. If, instead, we would decrease S_0^o, (A.5a) to (A.5c) would require $T^O < 0$, so that $O \to S \to L$ cannot occur if $S_0^o \le l_1(S_0^f)$. This proves (c) and (d) of part (i).

If $S_0^f > S_M^f$ the equilibrium cannot be $S \to L$ or $O \to S \to L$ because from (A.2a) to (A.2b) and (A.5a) to (A.5c) this would require $T^L < T^S$. Hence, if $S_0^f > S_M^f$ the only possibilities are $S \to F$ and $O \to S \to F$. At the boundary $S_0^f = l_2(S_0^f)$, (A.1a) to (A.1b) imply $q^f(0) = 0$ and (A.6a) to (A.6c) imply $T^O = 0$. Starting at this boundary, if we increase S_0^o, (A.1a) to (A.1b) would require $q^f(0) < 0$, so that $S \to F$ cannot occur if $S_0^o > l_2(S_0^f)$. If, instead, we would decrease S_0^o, (A.6a) to (A.6c) would require $T^O < 0$, so that $O \to S \to F$ cannot occur if $S_0^o \le l_2(S_0^f)$. This proves (a) and (b) of part (ii).

Finally, at $(S_0^f, S_0^o) = (S_M^f, S_M^o)$, if the equilibrium would read $S \to F$, (A.1a) to (A.1b) would imply $T^F = T^S$. If the equilibrium would read $S \to L$, (A.2a)-(A.2b) would imply $T^L = T^S$. Lemma 7 implies $T^S = \tilde{T}^S$. Fixing S_0^f at S_M^f and increasing S_0^o would require $q^f(0) < 0$ from (A.2a)- (A.2b) if the equilibrium would read $S \to L$, from (A.1a)- (A.1b) if the equilibrium would read $S \to F$, and from (13a)-(13c) if the equilibrium would read S. Hence, the only possibilities are $O \to S \to F$, $O \to S \to L$, and $O \to S$. But from (A.6a) to (A.6c) we get $T^F = T^S$ if $S_0^f = S_M^f$ and from (A.5a)-(A.5c) we obtain $T^L = T^S$ if $S_0^f = S_M^f$. Hence, the equilibrium reads $O \to S$. This proves (e) of part (i). □

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.jedc.2019.05.014

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